Mean Square Stabilization over Gaussian Finite-State Markov Channels

Liang Xu, Lihua Xie* and Nan Xiao

***Abstract*—The paper studies the mean square stabilization problem of discrete-time linear time-invariant systems over Gaussian finite-state Markov channels, which suffer from both signal-to-noise ratio constraint and correlated channel fading modeled by a Markov process. The existence of a fundamental limitation for mean square stabilization is firstly established. Then sufficient stabilization conditions under a Time Division Multiple Access (TDMA) communication schemes are derived in terms of the stability of a Markov Jump Linear System (MJLS). Moreover, we present a necessary and sufficient condition for mean square stabilization of two-dimensional systems controlled over power constrained Markov lossy channels. Furthermore, improved sufficient stabilization conditions are derived based on an adaptive TDMA communication scheme for general high-dimensional systems, which achieves a larger stabilization region than the TDMA communication scheme.**

I. Introduction

Networked control has been an active research topic in past decades [1], [2]. Until now, there are plentiful results revealing how communication channels affect the stability of networked control systems [3]–[8]. These results show that the stabilizability conditions are determined by unstable eigenvalues of the system matrix and channel parameters (in terms of data rate, signal-to-noise ratio, packet loss probability and so on).

Due to its ease of installation and maintenance, wireless communication has been widely used in networked control systems. However, since fading is unavoidable in wireless communications in urban, indoor and underwater environments [9], [10], in this paper we are interested in networked control over an analog channel that suffers from both signal-to-noise ratio constraint and time-varying channel fading. Previously, the case with independent and identically distributed (i.i.d.) channel fading has been studied in [8], [11]. However, the i.i.d. assumption fails to capture the correlation of channel conditions over time. Since Markov models are simple and effective in capturing temporal correlations of channel conditions [10], [12], [13], we are interested in the stabilization problem of discrete-time Linear Time Invariant (LTI) systems controlled over Gaussian finite-state Markov channels [14], where the channel fading is modeled by a time-homogeneous Markov process. Due to the existence of correlations of channel conditions over time, the methods used to deal with the i.i.d. channel fading in [8], [11] cannot be applied directly to the Markov channel fading case. Besides, the work [8] only considers the state feedback case and the plant under investigation is free of process and measurement noises. The output feedback case and how the plant noises affect the stabilizability of the networked control system have yet been studied.

In this paper, we propose observer/estimator designs and extend the channel resource allocation schemes in [8], [11] to the Gaussian Markov channel case and derive necessary and sufficient stabilization conditions by utilizing the stability of a Markov Jump Linear System (MJLS) and the i.i.d. property of the sojourn time of the Markov chain [15]. The contributions of this paper are three folds: firstly, it is shown that there exists a fundamental limitation for the mean square stabilization over Gaussian finite-state Markov channels; secondly, sufficient stabilization conditions under TDMA communication schemes are derived; thirdly, for power constrained Markov lossy channels, the necessary and sufficient stabilization condition is presented for two-dimensional systems and improved sufficient stabilization conditions are derived for general high-dimensional systems with an adaptive TDMA protocol.

The paper is organized as follows. The problem formulation and preliminaries are given in Sec. II. The existence of fundamental limitations for stabilization is demonstrated in Sec. III. Sufficient stabilization conditions for Gaussian finite-state Markov channels and power constrained Markov lossy channels are provided in Sec. IV and Sec. V, respectively. This paper ends with some concluding remarks in Sec. VI.

Notions. $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{N}^+$ are sets of real numbers, natural numbers and positive integers, respectively. $\rho(\cdot)$ denotes the spectral radius. $\mathfrak{e}$ represents the Euler's number. $\mathbb{E}\{\cdot\}$ is the expectation operator. $\mathbb{E}_y\{\cdot\}$ denotes the expectation conditioned on the event $Y = y$. $\{x_i\}_{i=0}^t$ represents the sequence $x_0, x_1, \ldots, x_t$ and is also written as x^t when the meaning is clear from the context.

II. Problem Formulation and Preliminaries

This paper studies the following discrete-time linear system

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + v_t, \\ y_t &= Cx_t + w_t, \end{aligned} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the system state; $y_t \in \mathbb{R}^m$ is the system output; $u_t \in \mathbb{R}$ is the control input; v_t, w_t are the process noise and measurement noise, respectively; (A, B) is stabilizable; (C, A) is observable; $\{v_t\}_{t\geq 0}$ and $\{w_t\}_{t\geq 0}$ are i.i.d. and with zero mean and bounded covariance matrices and are independent of the initial state x_0 which follows a zero

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mean Gaussian distribution with a bounded covariance matrix. Without loss of generality, we make the following assumption as in [16], [17].

Assumption 1: All the eigenvalues of A are either on or outside the unit circle.

This paper considers a networked control setting, where y_t is observed and encoded with the law $f_t(\cdot)$ and transmitted to the controller through a Gaussian Markov channel to generate the control signal u_t with the law $h_t(\cdot)$. The Markov channel corrupted with Gaussian noises is modeled as

$$r_t = \gamma_t s_t + \omega_t, \tag{2}$$

where s_t denotes the channel input satisfying an average power constraint, i.e., $\mathbb{E}\left\{s_t^2\right\} \leq P$; r_t is the channel output; γ_t is the channel fading which represents the variation of received signal power over time (also known as the channel state) and ω_t is an additive white Gaussian noise (AWGN) with zero-mean and bounded variance σ_ω^2. Different Markov models can be assumed for γ_t. In this paper, we are interested in two kinds of Gaussian Markov channels: the Gaussian finite-state Markov channel and the power constrained Markov lossy channel.

Gaussian Finite-State Markov Channels: The channel state $\{\gamma_t\}_{t\geq 0}$ is modeled as a time-homogeneous ergodic Markov process. γ_t takes values in a finite set of distinct non-negative values $\{\mathfrak{r}_1, \mathfrak{r}_2 \ldots, \mathfrak{r}_l\}$, which represents different fading levels [14]. The Markov transition probability matrix Q is defined by $Q = [q_{ij}]$ with

$$q_{ij} = \Pr\{\gamma_{t+1} = \mathfrak{r}_j | \gamma_t = \mathfrak{r}_i\}. \tag{3}$$

Power Constrained Markov Lossy Channels: The channel state $\{\gamma_t\}_{t\geq 0}$ is modeled as a Markov lossy process. γ_t only switches between two states: the state $\mathfrak{r}_1 = 0$ and the state $\mathfrak{r}_2 = 1$, where $\mathfrak{r}_1 = 0$ indicates the appearance of channel fading and the transmission fails and $\mathfrak{r}_2 = 1$ means that the channel is free of fading and the transmission is successful. Therefore, the Markov process has the following transition probability matrix

$$Q = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}, \tag{4}$$

where p represents the failure rate and q denotes the recovery rate. To avoid any trivial case, p and q are assumed to be strictly positive and less than 1, i.e., $0 < p, q < 1$, so that the Markov process is ergodic. The power constrained Markov lossy channel is one special kind of Gaussian finite-state Markov channels, and has several unique properties that allow to derive refined results compared to general finite-state Markov channels.

For both kinds of channels, we assume that $\{\omega_t\}_{t\geq 0}$ is i.i.d.; x_0, $\{v_t\}_{t\geq 0}$, $\{w_t\}_{t\geq 0}$, $\{\gamma_t\}_{t\geq 0}$ and $\{\omega_t\}_{t\geq 0}$ are independent; the channel state information is known at the receiver side and the channel output and the channel state are fed back to the transmitter through a noiseless feedback channel with one-step delay [8]. The feedback configuration and the information structure of the sensor and controller are depicted in Fig. 1.

Remark 1: The remote control setting in Fig. 1 has been widely adopted in networked control research (e.g., [17]–[19]). The aerial robotics research platform in [20] is one example of our feedback control configuration. The attitude and position of the aerial robot are observed via a sensing system such as a motion capture system. The observed value is processed on one or more standard computers and then transmitted to the aerial robot over wireless channels to implement the control algorithm.

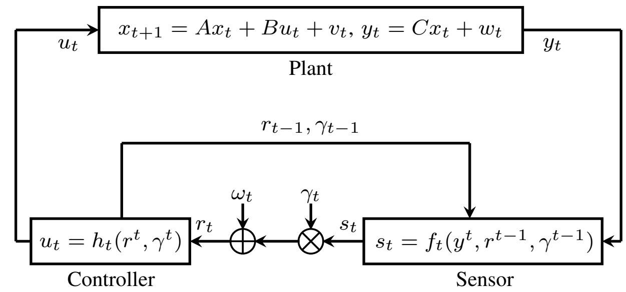

Fig. 1: Networked control over Gaussian Markov channels

Throughout the paper, a stochastic system with state x_t is mean square stable if $\sup_t \mathbb{E}\left\{x_t' x_t\right\} < \infty$. We try to characterize requirements on Gaussian finite-state Markov channels and power constrained Markov lossy channels such that there exist sensing and controlling strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$ which can stabilize the LTI dynamics (1). In following, we present several preliminary results that would be used in the subsequent analysis.

A. Stability of Markov Jump Linear Systems

Denote the instantaneous channel capacity as $c_t = \frac{1}{2}\ln(1 + \frac{\gamma_t^2 P}{\sigma_\omega^2})$. Since $\{\gamma_t\}_{t\geq 0}$ is Markovian, so is $\{c_t\}_{t\geq 0}$ and c_t takes values in a finite set $\{\mathfrak{c}_1, \ldots, \mathfrak{c}_l\}$ with $\mathfrak{c}_i = \frac{1}{2}\ln(1 + \frac{\mathfrak{r}_i^2 P}{\sigma_\omega^2})$ and is with the same Markov transition probability (3). Consider the MJLS defined by

$$z_{t+1} = \lambda^2 \mathfrak{e}^{-\frac{2}{o}c_t} z_t + a, \tag{5}$$

where $z_t \in \mathbb{R}$ with $z_0 < \infty$; $\lambda \in \mathbb{R}$; $o \in \mathbb{N}^+$; $a \geq 0$ and $\{c_t\}_{t\geq 0}$ is the Markov process described above. Let $H_o = Q' D_o$ with $D_o = \mathrm{diag}(\mathfrak{e}^{-\frac{2}{o}\mathfrak{c}_1}, \ldots, \mathfrak{e}^{-\frac{2}{o}\mathfrak{c}_l})$. Similar to Lemma 1 in [19], [21], we have the following necessary and sufficient condition characterizing the first moment stability of (5).

Lemma 1: The first moment of the system (5) is stable, i.e., $\sup_t \mathbb{E}\left\{|z_t|\right\} < \infty$, if and only if $\lambda^2 < \frac{1}{\rho(H_o)}$.

B. Sojourn Times for Markov Lossy Process

Associated with the Markov lossy process $\{\gamma_t\}_{t\geq 0}$, a stochastic time sequence $\{T_k\}_{k\geq 0}$ is introduced to denote the time at which the transmission is successful. Without loss of generality, let $\gamma_0 = \mathfrak{r}_2$ [18]. Then $T_0 = 0$ and T_k, $k \geq 1$ is precisely defined by

$$\begin{aligned} T_1 &= \inf\{k : k \geq 1, \gamma_k = 1\}, \\ T_2 &= \inf\{k : k \geq T_1, \gamma_k = 1\}, \\ &\vdots \quad \vdots \\ T_k &= \inf\{k : k \geq T_{k-1}, \gamma_k = 1\}. \end{aligned} \tag{6}$$

By the ergodic property of the Markov process $\{\gamma_k\}_{k\geq 0}, T_k$, $\forall k \in \mathbb{N}$ is finite almost everywhere (abbreviated as a.e.). Thus, the integer valued sojourn time $\{T_k^*\}_{k>0}$ which denotes the time duration between two successive successful transmissions is well-defined a.e., where

$$T_k^* = T_k - T_{k-1} > 0. \tag{7}$$

Moreover, we have the following characterization of the probability distribution of sojourn times $\{T_k^*\}_{k>0}$.

Lemma 2 ([15]): The sojourn times $\{T_k^*\}_{k>0}$ are i.i.d.. Furthermore, the distribution of T_k^* is explicitly expressed as

$$\Pr(T_k^* = i) = \begin{cases} 1-p & i = 1, \\ pq(1-q)^{i-2} & i > 1. \end{cases}$$

III. Fundamental Limitations

Let $\lambda_1, \ldots, \lambda_d$ be the distinct unstable eigenvalues (if λ_i is complex, its conjugate is excluded from this list) of A with $|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_d|$, and let m_i be the algebraic multiplicity of λ_i. The real Jordan canonical form J of A then has the block diagonal structure $J = \mathrm{diag}(J_1, \ldots, J_d) \in \mathbb{R}^{n\times n}$ [22], where the block $J_i \in \mathbb{R}^{\mu_i\times\mu_i}$ and $|\det J_i| = |\lambda_i|^{\mu_i}$, with $\mu_i = m_i$ if $\lambda_i \in \mathbb{R}$, and $\mu_i = 2m_i$ otherwise. It is clear that the mean square stability of (1) is equivalent to the mean square stability of

$$x_{t+1} = Jx_t + OBu_t + Ov_t, \tag{8}$$

$$y_t = CO^{-1}x_t + w_t, \tag{9}$$

for some invertible matrix O.

The following theorem characterizes a fundamental limitation for mean square stabilization over Gaussian finite-state Markov channels. The necessity is obtained via an information theoretic argument as in [8], but with differences due to the application of output feedback and the existence of process and measurement noises.

Theorem 1: There exist sensing and controlling strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$, such that the system (1) can be mean square stabilized over the Gaussian finite-state Markov channel only if $[|\lambda_1|, \ldots, |\lambda_d|]' \in \mathbb{R}^d$ satisfy

$$\left(\prod_{i=1}^{d} |\lambda_i|^{a_i o_i}\right)^{\frac{2}{o}} < \frac{1}{\rho(H_o)}, \tag{10}$$

for all $o_i \in \{0, \ldots, m_i\}$, $i = 1, \ldots, d$ with $o = \sum_{i=1}^{d} a_i o_i$, where $a_i = 1$ if $\lambda_i \in \mathbb{R}$ and $a_i = 2$ otherwise.

Proof: We use uppercase letters X, S, R, Γ to denote random variables of the system state, the channel input, the channel output and the channel fading. We use the lowercase letters x, s, r, γ to denote their realizations. Following a similar line of arguments as in [8], we can show that

$$N_{\gamma^t}(X_{t+1}|R^t) \geq (\det A)^{\frac{2}{n}} \mathfrak{e}^{-\frac{2}{n}c_t} N_{\gamma^{t-1}}(X_t|R^{t-1}), \tag{11}$$

where $N_\gamma(X|R)$ denotes the average conditional entropy power of X given the events $R = r$ and $\Gamma = \gamma$ and averaged only over R. In view of Proposition II.1 in [4], a necessary condition to ensure the mean square stability of X_t is that the first moment of $N_{\gamma^t}(X_{t+1}|R^t)$ should converge to zero asymptotically. Thus, the MJLS $z_{k+1} = (\det A)^{\frac{2}{n}} \mathfrak{e}^{-\frac{2}{n}c_t} z_k$ should be stable in the first moment. Following Lemma 1, a necessary condition to ensure the mean square stability can be obtained as

$$(\det A)^{\frac{2}{n}} < \frac{1}{\rho(H_n)}. \tag{12}$$

Notice that each block J_i has an invariant real subspace $\mathcal{A}_{o_i}$ of dimension $a_i o_i$, for any $o_i \in \{0, \ldots, m_i\}$. Consider the subspace $\mathcal{A}$ formed by taking the product of the invariant sub-spaces $\mathcal{A}_{o_i}$ for each real Jordan block. The total dimension of $\mathcal{A}$ is $\sum_{i=1}^{d} a_i o_i$ and the real Jordan form for the system matrix of the dynamics in the subspace $\mathcal{A}$ is $J^{\mathcal{V}}$ with $|\det J^{\mathcal{V}}| = \prod_{i=1}^{d} |\lambda_i|^{a_i o_i}$. Since (1) is mean square stabilizable, the dynamics in the subspace $\mathcal{A}$ is also mean square stabilizable. Following a similar line of arguments as in the derivation of (12), the fundamental limitation (10) can be obtained. ■

Let $\delta = \frac{\sigma_\omega^2}{P+\sigma_\omega^2}$. We can derive the necessity for control over power constrained Markov lossy channels from Theorem 1 directly. Firstly, the following lemma is need, whose proof is given in Appendix.

Lemma 3: Let Q be defined in (4); $D = \left[\begin{smallmatrix} 1 & 0 \\ 0 & \delta \end{smallmatrix}\right]$ with $0 < q, p, \delta < 1$; $\lambda \in \mathbb{R}$, $|\lambda| \geq 1$ and T_k^* be defined in (7). The following conditions are equivalent: (i) $\lambda^2 \rho(Q'D) < 1$, (ii) $\mathbb{E}\left\{\lambda^{2T_k^*}\right\}\delta < 1$, (iii)

$$1 - \lambda^2(1-q) > 0, \tag{13}$$

$$\lambda^2\delta\left[1 + \frac{p(\lambda^2 - 1)}{1 - \lambda^2(1-q)}\right] < 1. \tag{14}$$

The fundamental limitation for control over power constrained Markov lossy channels is stated below.

Theorem 2: There exist sensing and controlling strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$, such that the system (1) can be mean square stabilized over the power constrained Markov lossy channel only if $[|\lambda_1|, \ldots, |\lambda_d|]' \in \mathbb{R}^d$ satisfy

$$1 - \left(\prod_{i=1}^{d} |\lambda_i|^{a_i o_i}\right)^{\frac{2}{o}} (1-q) > 0, \tag{15}$$

$$\delta^{\frac{1}{o}} \left(\prod_{i=1}^{d} |\lambda_i|^{a_i o_i}\right)^{\frac{2}{o}} \left[1 + \frac{p(\left(\prod_{i=1}^{d} |\lambda_i|^{a_i o_i}\right)^{\frac{2}{o}} - 1)}{1 - (1-q)\left(\prod_{i=1}^{d} |\lambda_i|^{a_i o_i}\right)^{\frac{2}{o}}}\right] < 1, \tag{16}$$

for all $o_i \in \{0, \ldots, m_i\}$, $i = 1, \ldots, d$ with $o = \sum_{i=1}^{d} a_i o_i$.

Proof: Since

$$H_o = Q'D_o = \begin{bmatrix} 1-q & p \\ q & 1-p \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \delta^{\frac{1}{o}} \end{bmatrix},$$

for power constrained Markov lossy channels, in view of Theorem 1 and Lemma 3, the necessity can be obtained. ■

IV. Mean square stabilization over Gaussian finite-state Markov channels

In this section, we provide a sufficient stabilization condition for control over Gaussian finite-state Markov channels via the construction of observer, estimator, controller, channel encoder, decoder and scheduler. The observer/estimator/controller is reproduced from [19], [22], which mimics the optimal estimation and control scheme in LQG control [23]. The channel encoder/decoder/scheduler design is borrowed from [8], which adopts a TDMA scheme to transmit multiple sources over a scalar channel.

A. Communication Structure

The entire communication scheme is shown in Fig. 2. The observer and estimator can be regarded as the source encoder and decoder, which take the measurement signal y_t to estimate the system state $\hat{x}_t$. The channel encoder and decoder are designed to reliably transmit source signals over the uncertain channel. Since the observer/encoder is aware of the one-step delayed channel fading and channel output via the feedback link, it can thus simulate the decoder/estimator/controller to obtain the estimated state $\hat{x}_t$ and the control input u_t.

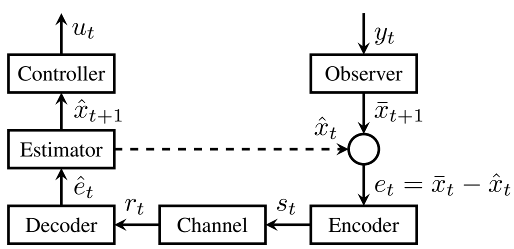

Fig. 2: Communication Structure

B. Observer/Estimator/Controller Design

The Luenberger observer is designed as

$$\bar{x}_{t+1} = A\bar{x}_t + Bu_t - L(y_t - C\bar{x}_t), \tag{17}$$

where $\bar{x}_0 = 0$ and L is selected such that $A+LC$ is Hurwitz. The estimator generates the estimate $\hat{x}_t$ with

$$\hat{x}_{t+1} = A\hat{x}_t + A\hat{e}_t + Bu_t, \tag{18}$$

where $\hat{x}_0 = 0$ and $\hat{e}_t$ is the output of the channel decoder. The controller is given by

$$u_t = K\hat{x}_t, \tag{19}$$

where K is selected such that $A + BK$ is Hurwitz. With the above observer, estimator and controller design, we have the following result.

Lemma 4: If there exists a pair of channel encoder and decoder, such that $\sup_t \mathbb{E}\left\{\|e_t\|^2\right\} < \infty$ with $e_t = \bar{x}_t - \hat{x}_t$, the system (1) is mean square stabilizable over the Gaussian finite-state Markov channel.

Proof: In view of (1) and (17), we have

$$x_{t+1} - \bar{x}_{t+1} = (A + LC)(x_t - \bar{x}_t) + v_t + Lw_t. \tag{20}$$

Since L is selected such that $A + LC$ is stable, we have $\sup_t \mathbb{E}\left\{\|x_t - \bar{x}_t\|^2\right\} < \infty$. From the observer dynamics (17) and the controller (19), we have $\bar{x}_{t+1} = (A + BK)\bar{x}_t - BK(\bar{x}_t - \hat{x}_t) - LC(x_t - \bar{x}_t) - Lw_t$. Since $A+BK$ is Hurwitz and $\sup_t \mathbb{E}\left\{\|x_t - \bar{x}_t\|^2\right\} < \infty$, if $\sup_t \mathbb{E}\left\{\|e_t\|^2\right\} < \infty$, we have $\sup_t \mathbb{E}\left\{\|\bar{x}_t\|^2\right\} < \infty$. Therefore, we have

$$\begin{aligned}\sup_t \mathbb{E}\left\{\|x_t\|^2\right\} &= \sup_t \mathbb{E}\left\{\|x_t - \bar{x}_t + \bar{x}_t\|^2\right\} \\ &\leq \sup_t \mathbb{E}\left\{\|x_t - \bar{x}_t\|^2\right\} + \sup_t \mathbb{E}\left\{\|\bar{x}_t\|^2\right\} < \infty,\end{aligned}$$

which implies that the original system (1) is mean square stable. The proof is completed. ■

In view of the above lemma, we are now to design channel encoder/decoder to ensure that $\sup_t \mathbb{E}\left\{\|e_t\|^2\right\} < \infty$. The dynamics for e_t is

$$e_{t+1} = A(e_t - \hat{e}_t) + \Phi_t, \tag{21}$$

where $\Phi_t = -LC(x_t - \bar{x}_t) - Lw_t$. From (20), we have that

$$x_t - \bar{x}_t = (A + LC)^t x_0 + \sum_{i=0}^{t-1}(A + LC)^{t-1-i}(v_i + Lw_i).$$

Since $x_0, \{v_t\}_{t\geq 0}, \{w_t\}_{t\geq 0}$ are independent and with zero mean and bounded variance, $x_t - \bar{x}_t$ and thus Φ_t are with zero mean and bounded variance.

C. Encoder/Decoder/Scheduler Design

To transmit the n-dimensional vector e_t through the scalar channel, the TDMA strategy is used. There are n encoder/decoder pairs to transmit the n sources $\{e_{1,t}, \ldots, e_{n,t}\}$ with $e_{i,t}$ being the i-th value of e_t and a scheduler to multiplex the channel use. Suppose at time t, the i-th encoder/decoder pair is scheduled to use the channel. The Encoder i first generates a symbol $s_{i,t}$, which is a scaled version of $e_{i,t}$ to satisfy the channel input power constraint, and transmits it to the decoder through the communication channel. The Decoder i then forms the minimal mean square error estimate $\hat{e}_{i,t}$ based on the channel output $r_{i,t}$. The estimator maintains an array $\hat{e}_t = [\hat{e}_{1,t}, \ldots, \hat{e}_{n,t}]'$ that represents the estimate of e_t, which is set to 0 at $t = 0$. When the information about $e_{i,t}$ is transmitted, only $\hat{e}_{i,t}$ is updated at the estimator side. The channel encoder/decoder/scheduler structure is illustrated in Fig. 3.

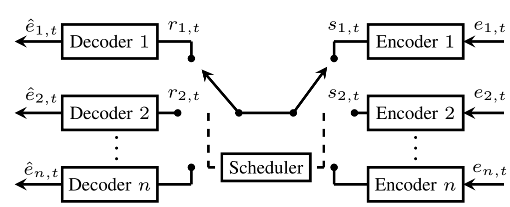

Fig. 3: Channel encoder/decoder/scheduler structure

If at time t, the Encoder i is scheduled to use the channel, then the encoder generates

$$s_{i,0} = 0, \quad s_{i,t} = \sqrt{\frac{P}{\sigma^2_{e_{i,t}}}} e_{i,t}, t \geq 1, \tag{22}$$

where $\sigma^2_{e_{i,t}}$ represents the variance of $e_{i,t}$. The Decoder i satisfies

$$\hat{e}_{i,t} = \frac{\mathbb{E}_{\gamma_t}\{r_{i,t}e_{i,t}\}}{\mathbb{E}_{\gamma_t}\{r^2_{i,t}\}} r_{i,t}. \tag{23}$$

It is clear from (21) and the designed communication scheme that $\mathbb{E}\{e_t\} = 0$ and $\mathbb{E}\{\hat{e}_t\} = 0$.

Remark 2: The observer/estimator/controller design and the encoder/decoder design are similar to those of [24], [25], where there is a filter that obtains the best state estimate at the transmitter side, and the transmitter sends only the innovation obtained as the difference between the best transmitter side state estimate and the best receiver side state estimate. The estimator/controller design in [24], [25] is optimal for scalar systems in terms of the LQ performance, while in this paper only stability is concerned. However, our work provides non-trivial extensions to vector systems.

The scheduling Algorithm 1 is designed, which adopts a TDMA strategy and allocates a fixed transmission period to each encoder/decoder pair, where $\tau_i, i = 1, \dots, n$ are scheduler parameters to be specified later. We assume that both the encoder and the decoder know the scheduling algorithm. Since the switching among transmissions is only determined by time, we do not need to consider the coordination among encoders and decoders.

Algorithm 1: TDMA Scheduler for n-dimensional Systems

In the k-th round of transmissions

- The first encoder/decoder pair is scheduled to use the channel for τ_1 times.
- ...
- The j-th encoder/decoder pair is scheduled to use the channel for τ_j times.
- ...
- The n-th encoder/decoder pair is scheduled to use the channel for τ_n times.
- Repeat this process.

D. Sufficient Stabilization Results

Theorem 3: If

$$\prod_{i=1}^{d} |\lambda_i|^{2\mu_i} < \frac{1}{\rho(H_1)}, \tag{24}$$

there exist τ_i, $i = 1, \dots, n$, such that the system (1) can be mean square stabilized over the Gaussian finite-state Markov channel with the proposed TDMA communication scheme.

In view of Lemma 4, if (21) is mean square stable, the system (1) can be mean square stabilized over the Gaussian finite-state Markov channel. Thus, the key in proving Theorem 3 is to show that there exist τ_is such that (21) is mean square stable. Moreover, with the designed TDMA communication scheme, we can show that each subsystem in (21) is described by a MJLS. If (24) holds, we have that $|\lambda_i|^{\frac{2\sum_j \ln|\lambda_j|}{\ln|\lambda_i|}} \rho(H_1) < 1$, for $i = 1, \dots, n$ (for the case that $\lambda_1, \dots, \lambda_d$ are real and $m_i = \mu_i = 1$). If τ_i is selected such that $\frac{\tau_i}{\sum_j \tau_j} = \frac{\ln|\lambda_i|}{\sum_j \ln|\lambda_j|}$, the MJLS is stable, which further implies the mean square stability of (21). Then the original system is mean square stable. The detailed proof is provided as below.

Proof: Without loss of generality, we assume that $\lambda_1, \dots, \lambda_d$ are real and $m_i = \mu_i = 1$. For other cases, the theorem can be proved by combining the following analysis with a similar line of arguments used in [26].

In the first step, we shall derive the dynamics for the mean square value of $e_{i,t}$. From (21), we obtain

$$e_{i,t+1} = \lambda_i(e_{i,t} - \hat{e}_{i,t}) + \Phi_{i,t}. \tag{25}$$

Analogous to the analysis in [11], we can show that with the encoder (22) and the decoder (23),

$$\mathbb{E}_{\gamma^{t+1}}\{e_{i,t+1}^2\} = \lambda_i^2 \mathfrak{e}^{-2c_t} \mathbb{E}_{\gamma^t}\{e_{i,t}^2\} + \mathbb{E}\left\{\Phi_{i,t}^2\right\}, \tag{26}$$

if the i-th encoder/decoder pair is scheduled to use the channel at time t. Let $\tau = \sum_{i=1}^{n} \tau_i$ and $\eta_{i,k\tau} = \mathbb{E}\{e_{1,k\tau}^2 | \gamma_{k\tau} = \mathfrak{r}_i\} \Pr(\gamma_{k\tau} = \mathfrak{r}_i)$. Since from time $k\tau + 1$ to $k\tau + \tau_1$, the first encoder/decoder pair is scheduled to use the channel from Algorithm 1, we have that

$$\begin{aligned}
\eta_{j,k\tau+1} &= \mathbb{E}\left\{e_{1,k\tau+1}^2 | \gamma_{k\tau+1} = \mathfrak{r}_j\right\} \Pr(\gamma_{k\tau+1} = \mathfrak{r}_j) \\
&= \sum_{i=1}^{l} \Pr(\gamma_{k\tau} = \mathfrak{r}_i | \gamma_{k\tau+1} = \mathfrak{r}_j) \Pr(\gamma_{k\tau+1} = \mathfrak{r}_j) \\
&\qquad \times \mathbb{E}\left\{e_{1,k\tau+1}^2 | \gamma_{k\tau+1} = \mathfrak{r}_j, \gamma_{k\tau} = \mathfrak{r}_i\right\} \\
&\overset{(a)}{=} \sum_{i=1}^{l} \Pr(\gamma_{k\tau+1} = \mathfrak{r}_j | \gamma_{k\tau} = \mathfrak{r}_i) \Pr(\gamma_{k\tau} = \mathfrak{r}_i) \\
&\qquad \times \mathbb{E}\left\{e_{1,k\tau+1}^2 | \gamma_{k\tau+1} = \mathfrak{r}_j, \gamma_{k\tau} = \mathfrak{r}_i\right\} \\
&\overset{(b)}{=} \sum_{i=1}^{l} q_{ij} \Pr(\gamma_{k\tau} = \mathfrak{r}_i) \mathbb{E}\left\{e_{1,k\tau+1}^2 | \gamma_{k\tau} = \mathfrak{r}_i\right\} \\
&\overset{(c)}{\le} \sum_{i=1}^{l} q_{ij} \Pr(\gamma_{k\tau} = \mathfrak{r}_i) \frac{\lambda_1^2}{\mathfrak{e}^{2\mathfrak{c}_i}} \mathbb{E}\left\{e_{1,k\tau}^2 | \gamma_{k\tau} = \mathfrak{r}_i\right\} + \mathbb{E}\left\{\Phi_{1,k\tau}^2\right\} \\
&= \sum_{i=1}^{l} \frac{\lambda_1^2}{\mathfrak{e}^{2\mathfrak{c}_i}} q_{ij} \eta_{i,k\tau} + \mathbb{E}\left\{\Phi_{1,k\tau}^2\right\},
\end{aligned}$$

where (a) follows from the Bayes law; (b) is due to the fact that $e_{1,k\tau+1}$ is independent with $\gamma_{k\tau+1}$ and (c) arises from (26). Let $\eta_{k\tau} = [\eta_{1,k\tau}, \eta_{2,k\tau}, \dots, \eta_{l,k\tau}]'$, then we have $\eta_{k\tau+1} \le \lambda_1^2 Q' D_1 \eta_{k\tau} + \mathbf{1}\mathbb{E}\left\{\Phi_{1,k\tau}^2\right\}$, where $\mathbf{1}$ is a vector with all elements to be one. With similar derivations we have that

$$\begin{aligned}
\eta_{k\tau+\tau_1} &\le \lambda_1^{2\tau_1} H_1^{\tau_1} \eta_{k\tau} \\
&\quad + \sum_{i=0}^{\tau_1-1} (\lambda_1^2 H_1)^{\tau_1-1-i} \mathbf{1}\mathbb{E}\left\{\Phi_{1,k\tau+i}^2\right\}.
\end{aligned} \tag{27}$$

Since from the time $k\tau + \tau_1 + 1$ to $(k+1)\tau$, there are no scheduled transmissions for the first encoder/decoder pair, similar to the derivation of (27), we have

$$\begin{aligned}
\eta_{(k+1)\tau} &\le \lambda_1^{2(\tau-\tau_1)} (Q')^{\tau-\tau_1} \eta_{k\tau+\tau_1} \\
&\quad + \sum_{i=0}^{\tau-\tau_1-1} (\lambda^2 Q')^{\tau-\tau_1-1-i} \mathbf{1}\mathbb{E}\left\{\Phi_{1,k\tau+\tau_1+i}^2\right\}.
\end{aligned} \tag{28}$$

Combining (27) and (28), we have that

$$\eta_{(k+1)\tau} \le \lambda_1^{2\tau} (Q')^{\tau-\tau_1} H_1^{\tau_1} \eta_{k\tau} + \Psi_{k\tau}, \tag{29}$$

where

$$\Psi_{k\tau} = \lambda_1^{2(\tau-\tau_1)}(Q')^{\tau-\tau_1}\sum_{i=0}^{\tau_1-1}(\lambda_1^2 H_1)^{\tau_1-1-i}1\mathbb{E}\left\{\Phi_{1,k\tau+i}^2\right\} + \sum_{i=0}^{\tau-\tau_1-1}(\lambda^2 Q')^{\tau-\tau_1-1-i}\mathbf{1}\mathbb{E}\left\{\Phi_{1,k\tau+\tau_1+i}^2\right\}$$

and $\Psi_{k\tau}$ is bounded.

In the second step, we will show that if the sufficient condition (24) is satisfied, there exist τ_is such that (29) is mean square stable. If (24) holds, we have $\ln\rho(H_1) + 2\sum_j \ln|\lambda_j| < 0$. Therefore, there exists $\varsigma > 0$, such that $\ln\rho(H_1) + 2\sum_j \ln|\lambda_j| + \varsigma = 0$, which also implies $2\ln|\lambda_i| + \alpha_i \ln\rho(H_1) = -\frac{\varsigma}{n} < 0$, with $\alpha_i = \frac{2\ln|\lambda_i| + \frac{\varsigma}{n}}{2\sum_j \ln|\lambda_j| + \varsigma} > 0$ and $\sum_i \alpha_i = 1$. Thus, we have $\lambda_i^2\rho(H_1)^{\alpha_i} < 1$ for all $i = 1,\ldots,n$. Let $\iota = \min_i(2\log_{\rho(H_1)}|\lambda_i| + \alpha_i) > 0$. Since for every $\alpha_i \in \mathbb{R}$, there exists a rational sequence $\{\beta_{i,k}\}_{k\geq 0}$, such that $\lim_{k\to\infty}\beta_{i,k} = \alpha_i$, we have $\lim_{k\to\infty}\frac{\beta_{i,k}}{\sum_j \beta_{j,k}} = \frac{\alpha_i}{\sum_j \alpha_j} = \alpha_i$. Therefore, for the given ι, there exists $M \in \mathbb{N}^+$, such that $|\frac{\beta_{i,M}}{\sum_j \beta_{j,M}} - \alpha_i| < \iota$. Let $\vartheta_i = \frac{\sum_j \beta_{j,M}}{\beta_{i,M}}$. Then $\vartheta_i^{-1} > \alpha_i - \iota \geq -2\log_{\rho(H_1)}|\lambda_i|$. Thus, we have $\lambda_i^{2\vartheta_i}\rho(H_1) < 1$. In view of Lemma 5.6.10 in [27], there exists a norm $\|\cdot\|$ such that $\kappa_i := \|\lambda_i^{2\vartheta_i}H_1\| < 1$. From the equivalence of norms, we have that $\|\cdot\| \leq \epsilon\|\cdot\|_1$ for some $\epsilon > 1$. Then $\tau_i \in \mathbb{N}^+$ is selected to satisfy that $\tau_i > -\log_{\kappa_i}\epsilon$ and $\beta_{i,M} = \frac{\tau_i}{\bar{\tau}}$ for all $i = 1,\ldots,n$ and for some $\bar{\tau}$. The existence of such τ_is can always be guaranteed by firstly writing rational numbers $\beta_{i,M}$s into fractions and then reducing fractions to a common denominator and finally scaling the numerators and denominators simultaneously to obtain a sufficiently large numerator τ_i which satisfies $\tau_i > -\log_{\kappa_i}\epsilon$. Then we have from (29) that

$$\begin{aligned}\|\eta_{(k+1)\tau}\| &\leq \|(Q')^{\tau-\tau_1}\|\|\lambda_1^{2\tau}H_1^{\tau_1}\|\|\eta_{k\tau}\| + \|\Psi_{k\tau}\| \\ &\leq \kappa_1^{\tau_1}\|(Q')^{\tau-\tau_1}\|\|\eta_{k\tau}\| + \|\Psi_{k\tau}\| \\ &\leq \kappa_1^{\tau_1}\epsilon\|(Q')^{\tau-\tau_1}\|_1\|\eta_{k\tau}\| + \|\Psi_{k\tau}\| \\ &\leq \kappa_1^{\tau_1}\epsilon\|\eta_{k\tau}\| + \|\Psi_{k\tau}\|.\end{aligned}$$

Since $\kappa_1^{\tau_1}\epsilon < 1$, we know that $\|\eta_{k\tau}\|$ is bounded. Since $\mathbb{E}\left\{e_{1,k\tau}^2\right\} = \sum_i^l \eta_{i,k\tau}$, we further have that $\mathbb{E}\left\{e_{1,k\tau}^2\right\}$ is bounded.

Similarly, we can also prove that $\sup_k \mathbb{E}\left\{e_{i,k\tau}^2\right\} < \infty$ for all $i = 2,\ldots,n$. Therefore, e_t is mean square bounded. In view of Lemma 4, the closed-loop system is mean square stable. The proof is completed. ■

Remark 3: Suppose $q_{ij} = q_j$ for $i,j = 1,\ldots,l$, then the Gaussian finite-state Markov channel degenerates to the power constrained fading channel [8] with finite i.i.d. channel states. The stabilization condition in Theorem 3 becomes $\prod_{i=1}^d |\lambda_i|^{2\mu_i}(\sum_{i=1}^l q_i\frac{\sigma_\omega^2}{\sigma_\omega^2 + \mathfrak{r}_i^2 P}) < 1$, which coincides with the result in [8].

The sufficient condition is also necessary for scalar systems as shown in the following corollary.

Corollary 1: Suppose $A = \lambda_1$ with $\lambda_1 \in \mathbb{R}$ and $|\lambda_1| \geq 1$. There exist sensing and controlling strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$, such that the system (1) can be mean square stabilized over the Gaussian finite-state Markov channel if and only if $\lambda_1^2 < \frac{1}{\rho(H_1)}$.

Generally, there exists a gap between the necessity (10) and the sufficiency (24) for high dimensional systems. In the next section, we will study power constrained Markov lossy channels and derive improved results.

V. Mean Square Stabilization over Power Constrained Markov Lossy Channels

In this section, by utilizing the properties of the Markov lossy process, we propose communication scheduling algorithms for power constrained Markov lossy channels and show that they can achieve a larger stabilization region than that with the TDMA scheduler. We first start with two-dimensional systems.

A. Two-dimensional Systems

The necessary and sufficient condition to ensure the mean square stability for two-dimensional systems controlled over power constrained Markov lossy channels is stated in the following theorem.

Theorem 4: Suppose $n = 2$. There exist sensing and controlling strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$, such that the system (1) can be mean square stabilized over the power constrained Markov lossy channel if and only if (15) and (16) hold.

For the case of two-dimensional systems with eigenvalues of equal magnitude, the communication scheme designed in Section V-B is shown to be optimal (in the sense that it achieves the largest stabilization region indicated by the necessary condition in Theorem 2); see Corollary 2. In this subsection, we only provide the optimal communication scheme for two-dimensional systems with eigenvalues having different magnitudes, i.e., $A = \left[\begin{smallmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{smallmatrix}\right]$ with $\lambda_1,\lambda_2 \in \mathbb{R}$ and $|\lambda_1| > |\lambda_2| \geq 1$. In view of Theorem 4, we only need to prove that the following conditions are sufficient

$$(1-q)\lambda_1^2 < 1, \tag{30}$$

$$\delta\lambda_1^2\left[1 + \frac{p(\lambda_1^2 - 1)}{1-(1-q)\lambda_1^2}\right] < 1, \tag{31}$$

$$\delta^{\frac{1}{2}}|\lambda_1\lambda_2|\left[1 + \frac{p(|\lambda_1\lambda_2| - 1)}{1-(1-q)|\lambda_1\lambda_2|}\right] < 1. \tag{32}$$

1) Optimal Scheduler Design: The communication structure is designed similarly as in Sec. IV with the same observer/estimator/controller design and the channel encoder/decoder design. The scheduling Algorithm 2 is then proposed, where $\phi = 2\frac{\ln|\lambda_1| - \ln|\lambda_2|}{\ln\delta}$ and τ_1 is the scheduler parameter to be specified later. Since the switching among transmissions in Algorithm 2 relies on the channel state information, which is known to the decoder and the encoder via the channel feedback, we do not need to consider the coordination among encoders and decoders. Algorithm 2 is based on the optimal scheduling algorithm for control over power constrained lossy channels [11], where it is shown that such allocation of channel resources is optimal for the stabilization of two-dimensional systems with i.i.d. channel states. Even though the channel state $\{\gamma_t\}_{t\geq 0}$ is correlated over time for the power constrained Markov lossy channel, the sojourn time $\{T_k^*\}_{k>0}$ is i.i.d.. We may study the channel from the perspective of the i.i.d. sojourn time sequence and expect that the Algorithm 2 is optimal as well.

Algorithm 2: Optimal Scheduler for Two-dimensional Systems

In the k-th round,

- The first encoder/decoder pair is scheduled to use the channel until the transmissions succeed for τ_1 times. Denote the time period to achieve this object as T_k^1.
-
  - If
  $$T_k^1 < -\frac{\tau_1}{\phi}, \tag{33}$$
  the second encoder/decoder pair is scheduled to use the channel until the transmissions succeed for $\tau_{2,k}$ times with
  $$\tau_{2,k} > \tau_1 + (T_k^1 + T_k^2)\phi, \tag{34}$$
  where T_k^2 denotes the minimal period of achieving this object.
  - Otherwise, set $T_k^2 = 0$ and do not conduct any transmissions.
- Repeat.

To make notions clear, we plotted the scheduled transmissions and the first round transmission in Fig. 4 and Fig. 5, respectively, where the definitions of T_k, T_k^*, T_k^1, T_k^2 and the new symbols $\bar{T}_k, \check{T}_k$ are summarized in Table I. It is clear from Algorithm 2 that $\bar{T}_i$ and $\bar{T}_j$ are i.i.d.; T_i^2 is independent of T_j^2 for any $i \neq j$. Besides, we have $T_1^1 = T_1^* + \ldots + T_{\tau_1}^*$, $T_1^2 = T_{\tau_1+1}^* + \ldots + T_{\tau_1+\tau_{2,1}}^*$.

$T_k, k \geq 0$	the time when the transmission is successful as defined in (6)
$T_k^*, k \geq 1$	time duration between two successive successful transmissions as defined in (7)
$T_k^1, k \geq 1$	the period to transmit the first encoder/decoder pair in the k-th round transmission
$T_k^2, k \geq 1$	the period to transmit the second encoder/decoder pair in the k-th round transmission
$\bar{T}_k, k \geq 1$	the total time to complete the k-th round of transmissions, i.e., $\bar{T}_k = T_k^1 + T_k^2$
$\check{T}_k, k \geq 0$	the time when k rounds of transmissions are completed, i.e., $\check{T}_k = \sum_{j=1}^k \bar{T}_j$

TABLE I: Lists of transmission related definitions

2) Scheduler Parameter Selection: If (30) holds, we have

$$\mathbb{E}\left\{\lambda_1^{2T_1^*}\right\} = \lambda_1^2[1 + \frac{p(\lambda_1^2 - 1)}{1-(1-q)\lambda_1^2}] > 1.$$

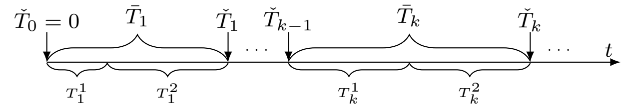

Fig. 4: Scheduled transmissions with Algorithm 2

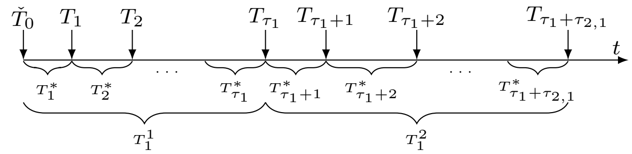

Fig. 5: The first round transmission with Algorithm 2

Since $(1-q)|\lambda_1\lambda_2| < 1$ from (30), if (32) holds, we have

$$\delta^{\frac{1}{2}}\mathbb{E}\left\{|\lambda_1\lambda_2|^{T_1^*}\right\} = \delta^{\frac{1}{2}}|\lambda_1\lambda_2|[1 + \frac{p(|\lambda_1\lambda_2| - 1)}{1-(1-q)|\lambda_1\lambda_2|}] < 1.$$

Since $\mathbb{E}\{\mathfrak{e}^{\theta + bT_1^*}\}$ with $b = 2\ln|\lambda_1| - \phi\theta$ is increasing in θ; when $\theta = 0$, $\mathbb{E}\{\mathfrak{e}^{\theta+bT_1^*}\} = \mathbb{E}\{\lambda_1^{2T_1^*}\} > 1$ and when $\theta = \frac{1}{2}\ln\delta$, $\mathbb{E}\{\mathfrak{e}^{\theta+bT_1^*}\} = \delta^{\frac{1}{2}}\mathbb{E}\{|\lambda_1\lambda_2|^{T_1^*}\} < 1$, we know that there exists θ^* with $\frac{1}{2}\ln\delta < \theta^* < 0$, such that

$$\mathbb{E}\left\{\mathfrak{e}^{\theta^* + bT_1^*}\right\} = 1. \tag{35}$$

The scheduler parameter τ_1 is then selected to satisfy

$$\tau_1 > \max\left\{\frac{-2\ln 2 + \theta^*(1-\phi)}{\ln\delta - 2\theta^*}, \frac{-\ln 2}{\ln\left(\delta\lambda_1^2[1 + \frac{p(\lambda_1^2-1)}{1-(1-q)\lambda_1^2}]\right)}\right\}. \tag{36}$$

3) Sufficiency Proof of Theorem 4: Firstly, we can prove that, if (31) holds and τ_1 is selected to satisfy (36), we have that

$$\mathbb{E}\left\{\lambda_1^{2\bar{T}_1}\delta^{\tau_1}\right\} < 1, \quad \mathbb{E}\left\{\lambda_2^{2\bar{T}_1}\delta^{\tau_{2,1}}\right\} < 1. \tag{37}$$

The proof follows the same steps as in the proof of Lemma 3 in [11]. Due to space limitations, we omit the proof for brevity.

Next, we will show that the randomly sampled sequence $\mathbb{E}\left\{e_{1,\check{T}_k}^2\right\}, k \geq 0$ is bounded. Conditioned on the sequence $\{\gamma_{\check{T}_{k-1}}, \gamma_{\check{T}_{k-1}+1}, \ldots, \gamma_{\check{T}_{k-1}+\bar{T}_k}\}$ and from (26), we have

$$\begin{aligned}
\mathbb{E}\left\{e_{1,\check{T}_k}^2\right\} &= \mathbb{E}\left\{e_{1,\check{T}_{k-1}+\bar{T}_k}^2\right\} = \prod_{j=0}^{\bar{T}_k-1}\lambda_1^2\delta^{\gamma_{\check{T}_{k-1}+j}}\mathbb{E}\left\{e_{1,\check{T}_{k-1}}^2\right\} \\
&\quad + \sum_{i=0}^{\bar{T}_k-1}\prod_{j=i+1}^{\bar{T}_k-1}\lambda_1^2\delta^{\gamma_{\check{T}_{k-1}+j}}\mathbb{E}\left\{\Phi_{1,\check{T}_{k-1}+i}^2\right\} \\
&= \lambda_1^{2\bar{T}_k}\delta^{\tau_1}\mathbb{E}\left\{e_{1,\check{T}_{k-1}}^2\right\} + \sum_{i=0}^{\bar{T}_k-1}\prod_{j=i+1}^{\bar{T}_k-1}\lambda_1^2\delta^{\gamma_{\check{T}_{k-1}+j}}\mathbb{E}\left\{\Phi_{1,\check{T}_{k-1}+i}^2\right\} \\
&\overset{(a)}{\leq} \lambda_1^{2\bar{T}_k}\delta^{\tau_1}\mathbb{E}\left\{e_{1,\check{T}_{k-1}}^2\right\} + \sum_{i=0}^{\bar{T}_k-1}\lambda_1^{2(\bar{T}_k-i-1)}\mathbb{E}\left\{\Phi_{1,\check{T}_{k-1}+i}^2\right\} \\
&\leq \lambda_1^{2\bar{T}_k}\delta^{\tau_1}\mathbb{E}\left\{e_{1,\check{T}_{k-1}}^2\right\} + \sup_t\mathbb{E}\left\{\Phi_{1,t}^2\right\}\sum_{i=0}^{\bar{T}_k-1}\lambda_1^{2(\bar{T}_k-i-1)}
\end{aligned}$$

$$\leq \lambda_1^{2\bar{T}_k}\delta^{\tau_1}\mathbb{E}\left\{e^2_{1,\check{T}_{k-1}}\right\} + \sup_t \mathbb{E}\left\{\Phi^2_{1,t}\right\}\frac{\lambda_1^{2(\bar{T}_k-1)}-\lambda_1^{-2}}{1-\lambda_1^{-2}},$$

where (a) follows from the fact that $\delta^{\gamma_k}\leq 1$ for any k. Thus, we have that

$$\mathbb{E}\left\{e^2_{1,\check{T}_k}\right\}\leq \mathbb{E}\left\{\lambda_1^{2\bar{T}_k}\delta^{\tau_1}\right\}\mathbb{E}\left\{e^2_{1,\check{T}_{k-1}}\right\} + \sup_t \mathbb{E}\left\{\Phi^2_{1,t}\right\}\mathbb{E}\left\{\frac{\lambda_1^{2(\bar{T}_k-1)}-\lambda_1^{-2}}{1-\lambda_1^{-2}}\right\}. \quad (38)$$

Since $\{\bar{T}_k\}_{k\geq 1}$ are i.i.d., we have $\mathbb{E}\{\lambda_1^{2\bar{T}_k}\delta^{\tau_1}\}<1$ and $\sup_t\mathbb{E}\{\Phi^2_{1,t}\}\mathbb{E}\{\frac{\lambda_1^{2(\bar{T}_k-1)}-\lambda_1^{-2}}{1-\lambda_1^{-2}}\}$ is bounded from (37), then the randomly sampled sequences $\mathbb{E}\left\{e^2_{1,\check{T}_k}\right\}, k\geq 0$ is bounded from (38). Similarly, we can also prove that $\mathbb{E}\left\{e^2_{2,\check{T}_k}\right\}, k\geq 0$ is bounded.

For any t, there must exist k such that $t\in[\check{T}_k,\check{T}_{k+1}]$. Thus, conditioned on the lossy process $\{\gamma_t\}_{t\geq 0}$, we obtain that for $i=1,2$,

$$\begin{aligned}\mathbb{E}\left\{e^2_{i,t}\right\} &= \prod_{j=\check{T}_k}^{t-1}\lambda_i^2\delta^{\gamma_j}\mathbb{E}\left\{e^2_{i,\check{T}_k}\right\} + \sum_{i=0}^{t-\check{T}_k-1}\prod_{j=\check{T}_k+i+1}^{t-1}\lambda_i^2\delta^{\gamma_j}\mathbb{E}\left\{\Phi^2_{i,\check{T}_k+i}\right\}\\ &\leq \lambda^{2(\check{T}_{k+1}-\check{T}_k-1)}\mathbb{E}\left\{e^2_{i,\check{T}_k}\right\} + \sup_t\mathbb{E}\left\{\Phi^2_{i,t}\right\}\frac{\lambda_i^{2(\check{T}_{k+1}-\check{T}_k-1)}-\lambda_i^{-2}}{1-\lambda_i^{-2}}\\ &\leq \lambda^{2(\bar{T}_k-1)}\mathbb{E}\left\{e^2_{i,\check{T}_k}\right\} + \sup_t\mathbb{E}\left\{\Phi^2_{i,t}\right\}\frac{\lambda_i^{2(\bar{T}_k-1)}-\lambda_i^{-2}}{1-\lambda_i^{-2}}.\end{aligned}$$

Thus, we have

$$\mathbb{E}\left\{e^2_{i,t}\right\}\leq \mathbb{E}\left\{\lambda^{2(\bar{T}_k-1)}\right\}\mathbb{E}\left\{e^2_{i,\check{T}_k}\right\} + \sup_t\mathbb{E}\left\{\Phi^2_{i,t}\right\}\mathbb{E}\left\{\frac{\lambda_i^{2(\bar{T}_k-1)}-\lambda_i^{-2}}{1-\lambda_i^{-2}}\right\}.$$

Since $\mathbb{E}\left\{\lambda_i^{2\bar{T}_k}\right\}$ and $\mathbb{E}\left\{e^2_{i,\check{T}_k}\right\}$ are bounded, we know that $\mathbb{E}\left\{e^2_{i,t}\right\}$ is bounded. In view of Lemma 4, the sufficiency is proved. ■

B. High-dimensional Systems

The key difficulty in stabilizing multi-dimensional systems over fading channels is to optimally allocate channel resources among different sub-dynamics. We can show that the desired optimal allocation is determined by the magnitudes of eigenvalues and the realization of the channel fading [8]. To optimally schedule the current transmission, we need to know the future fading realizations as shown in [8], which is not available due to the casualty constraint. For two-dimensional systems, we can adopt Algorithm 2 to overcome this problem, which first allocates a constant amount of channel resources to the first sub-dynamics and then optimally stops the transmissions for the second sub-dynamics. But this method is not applicable to three or higher dimensional systems since to optimally stop the transmissions for the second or subsequent sub-dynamics, we need the information of the channel fading realizations from the future transmissions for all the sub-dynamics, which is not possible due to the causal availability of the channel state information. In this subsection, an adaptive TDMA scheduling algorithm is proposed for high-dimensional systems, which is adaptive to the lossy process and outperforms the scheduling Algorithm 1 as shown later. The adaptive TDMA scheduler is stated in Algorithm 3, where $\tau_1,\ldots,\tau_n$ are scheduler parameters to be specified later.

Algorithm 3: Adaptive TDMA Scheduler for n-dimensional Systems

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- The first encoder/decoder pair is scheduled to use the channel, until the transmissions succeed for τ_1 times.
- The second encoder/decoder pair is scheduled to use the channel, until the transmissions succeed for τ_2 times.
- …
- The n-th encoder/decoder pair is scheduled to use the channel, until the transmissions succeed for τ_n times.
- Repeat.

Let T_k^i denote the period for the i-th encoder/decoder pair to achieve τ_i successful transmissions in the k-th round and define $\bar{T}_k$, $\check{T}_k$ analogously as in Subsection V-A. It is clear that T_k^i is independent with T_k^j, and $\bar{T}_i$ and $\bar{T}_j$ are i.i.d. for any $i\neq j$. A sufficient stabilization result with Algorithm 3 is stated in the following theorem.

Theorem 5: There exist sensing and controlling strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$, such that the system (1) can be mean square stabilized over the power constrained Markov lossy channel, if there exist α_i, $i=1,\ldots,d$ with $0<\alpha_i\leq 1$ and $\sum_{i=1}^d\alpha_i=1$ such that

$$(1-q)|\lambda_1|^2<1, \quad (39)$$

$$\delta^{\frac{\alpha_i}{\mu_i}}|\lambda_i|^2[1+\frac{p(|\lambda_i|^2-1)}{1-(1-q)|\lambda_i|^2}]<1, \quad (40)$$

for all $i=1,\ldots,d$.

Proof: Here we only consider the case that $\lambda_1,\ldots,\lambda_d$ are real and $m_i=\mu_1=1$. We can easily extend the analysis to other cases by combining the following analysis with similar arguments used in [26]. In view of Lemma 3, the sufficient condition in Theorem 5 is equivalent to the following condition

$$\mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}\delta^{\alpha_i}<1, \quad i=1,\ldots,n. \quad (41)$$

Let $\iota=\min_i(\log_\delta\mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}+\alpha_i)$. For any α_i, there exists a rational sequence $\{\beta_{i,k}\}_{k\geq 0}$, such that $\lim_{k\to\infty}\beta_{i,k}=\alpha_i$. Then $\lim_{k\to\infty}\frac{\beta_{i,k}}{\sum_j\beta_{j,k}}=\frac{\alpha_i}{\sum_j\alpha_j}=\alpha_i$. Therefore, for the given ι, there exists $M\in\mathbb{N}^+$, such that $|\frac{\beta_{i,M}}{\sum_j\beta_{j,M}}-\alpha_i|<\iota$ for all

$i = 1, \ldots, n$. Thus, $\frac{\beta_{i,M}}{\sum_j \beta_{j,M}} > \alpha_i - \iota \geq -\log_\delta \mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}$, which implies

$$\mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}\delta^{\frac{\beta_{i,M}}{\sum_j \beta_{j,M}}} < 1, \quad i = 1, \ldots, n.$$

Since $\beta_{1,M}, \ldots, \beta_{n,M}$ are rational, there exist integers $\tau_1, \ldots, \tau_n, \bar{\tau}$ such that $\beta_{i,M} = \frac{\tau_i}{\bar{\tau}}$ and $\mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}\delta^{\frac{\tau_i}{\sum_j \tau_j}} < 1$ for $i = 1, \ldots, n$, which implies

$$\mathbb{E}\left\{\lambda_i^{2\bar{T}_1}\delta^{\tau_i}\right\} = \mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}^{\tau_1 + \ldots + \tau_n}\delta^{\tau_i} < 1.$$

Similar to the proof of Theorem 4, we can then show that the sampled sequence $\mathbb{E}\left\{e^2_{i,\tilde{T}_k}\right\}$ is bounded, and further $\mathbb{E}\left\{e^2_{i,t}\right\}$ is bounded. In view of Lemma 4, the sufficiency is proved. ■

Remark 4: In view of Lemma 3, Theorem 5 can be equivalently stated as: if there exist α_is with $0 < \alpha_i \leq 1$ and $\sum_{i=1}^d \alpha_i = 1$, such that

$$\mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}^{\frac{\mu_i}{\alpha_i}}\delta < 1, \tag{42}$$

for $i = 1, \ldots, d$, the system is mean square stabilizable. Then the existence of α_is in Theorem 5 can be determined as follows. Let $\alpha_i^* = -\mu_i \log_\delta \mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}$, which is the lower bound for any feasible α_i from (42). If $\sum_i \alpha_i^* > 1$, there are no feasible α_is. Otherwise, one admissible α_i is given by $\alpha_i = \frac{\alpha_i^*}{\sum_j \alpha_j^*}$.

Remark 5: Theorem 3 can be equivalently expressed as: if there exist α_is with $0 < \alpha_i \leq 1$ and $\sum_{i=1}^d \alpha_i = 1$, such that

$$\lambda_i^{\frac{2\mu_i}{\alpha_i}}\rho(Q'D_1) < 1, \tag{43}$$

for $i = 1, \ldots, d$, the system is mean square stabilizable. For power constrained Markov lossy channels, in view of Lemma 3, (43) is equivalent to

$$\mathbb{E}\left\{\lambda_i^{\frac{\mu_i}{\alpha_i}2T_1^*}\right\}\delta < 1. \tag{44}$$

Since $\mathbb{E}\left\{\lambda_i^{2T_1^*}\right\}^{\frac{\mu_i}{\alpha_i}} \leq \mathbb{E}\left\{\lambda_i^{\frac{\mu_i}{\alpha_i}2T_1^*}\right\}$ from Jensen's inequality, any λ_i that satisfies (44), must also satisfy (42). Thus, the adaptive TDMA scheduler outperforms the TDMA scheduler in the sense that it can tolerate more unstable systems.

When all the strictly unstable eigenvalues have the same magnitude, the sufficient condition in Theorem 5 coincides with the necessary condition in Theorem 2, as shown in the following corollary.

Corollary 2: Suppose $|\lambda_1| = \cdots = |\lambda_{d_u}| = \tilde{\lambda} > 1$ and $|\lambda_{d_u+1}| = \cdots = |\lambda_d| = 1$ with $1 \leq d_u \leq d$. There exist encoding and decoding strategies $\{f_t(\cdot)\}_{t\geq 0}, \{h_t(\cdot)\}_{t\geq 0}$, such that the system (1) can be mean square stabilized over the power constrained Markov lossy channel if and only if

$$(1-q)\bar{\lambda}^2 < 1,$$
$$\delta^{\frac{1}{\mu_1 + \ldots + \mu_{d_u}}}\bar{\lambda}^2[1 + \frac{p(\bar{\lambda}^2 - 1)}{1 - (1-q)\bar{\lambda}^2}] < 1.$$

Remark 6: As an application of the derived theorems, we have the following extensions.

- When $p = 0, q = 1$, the power constrained Markov lossy channel degenerates to the AWGN channel, a necessary and sufficient condition to ensure mean square stabilizability from Theorem 2 and Theorem 5 is $\sum_i \mu_i \ln|\lambda_i| < \frac{1}{2}\ln(1 + \frac{P}{\sigma_\omega^2})$, which coincides with the stabilizability condition over AWGN channels in [3], [4].
- If $p = 1 - q$, we can obtain the stabilizability condition over power constrained lossy channels [11]. We can show that Theorem 2, Theorem 4 and Theorem 5 recover Lemma 1, Theorem 2 and Theorem 1 in [11], respectively.
- For the power constrained Markov lossy channel, taking the limit $P \to \infty$, $\sigma_\omega^2 \to 0$, we obtain the stabilizability condition for control over Markovian packet loss channel from Theorem 2 and Theorem 5 as $(1-q)|\lambda_1|^2 < 1$, which recovers the results in [15], [28], [29]. Moreover, if $p = 1 - q$, we can further recover the stabilization condition for control over i.i.d. erasure channels as in [5], [6].

C. Numerical Illustrations

For two-dimensional systems controlled over power constrained Markov lossy channels, suppose $P = 3, \sigma_\omega^2 = 1$, the regions for $(\ln|\lambda_1|, \ln|\lambda_2|)$ indicated by the derived necessary conditions and sufficient conditions are plotted in Fig. 6 under different failure and recovery rates. We plot the necessary stabilization region and sufficient stabilization regions achieved with the optimal scheduler, the TDMA scheduler and the adaptive TDMA scheduler for the case $p = 0.3, q = 0.6$. For the cases of $p = 0.6, q = 0.6$ and $p = 0.3, q = 0.9$, only the stabilization region indicated by the necessity and sufficiency with the optimal scheduler is plotted. The other sufficient stabilization regions are omitted for clarity but can be plotted in a similarly way as in the case of $p = 0.3, q = 0.6$.

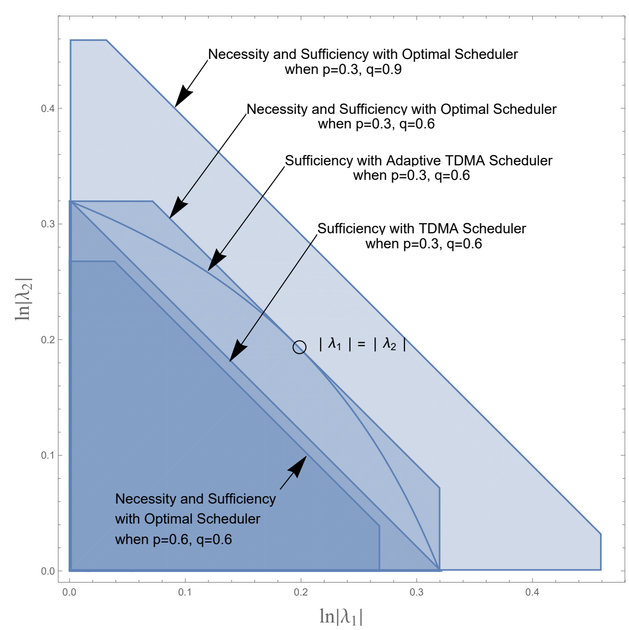

Fig. 6: Stabilization regions for $(\ln|\lambda_1|, \ln|\lambda_2|)$

For the given failure and recovery rate, it is clear that the adaptive TDMA scheduler achieves a larger stabilization region than the TDMA scheduler. When the two eigenvalues are with equal magnitude, the adaptive TDMA scheduler is optimal, which is implied in Corollary 2. Besides, the optimal scheduling Algorithm 2 is tight as proved in Theorem 4. Moreover, when we increase the failure rate p or the recovery rate q, the stabilization region is reduced or enlarged as expected due to the change of the reliability of the communication channel.

VI. Conclusions

This paper studies the mean square stabilization problem over Gaussian finite-state Markov channels. Necessary and sufficient conditions are derived for general Gaussian finite-state Markov channels. Improved sufficient conditions are presented for power constrained Markov lossy channels. The results imply that there exists a fundamental limitation for the mean square stabilization of networked control over Gaussian Markov channels. The proposed communication structure and the scheduler design also shed light on practical networked control system design. Further work will be devoted to reducing the gap between the necessary condition and the sufficient condition for general high-dimensional systems.

Appendix

Proof: (ii)$\leftrightarrow$(iii): In view of the probability distribution of T_k^* in Lemma 2, we have

$$\begin{aligned}\mathbb{E}\left\{\lambda^{2T_k^*}\right\} &= \sum_{i=1}^{\infty}\Pr(T_k^*=i)\lambda^{2i}\\ &= \Pr(T_k^*=1)\lambda^2 + \sum_{i=2}^{\infty}\Pr(T_k^*=i)\lambda^{2i}\\ &= (1-p)\lambda^2 + \sum_{i=2}^{\infty}pq(1-q)^{i-2}\lambda^{2i}.\end{aligned}$$

To guarantee the boundedness of $\mathbb{E}\left\{\lambda^{2T_k^*}\right\}$, we should have $\lambda^2(1-q)<1$. Then $\mathbb{E}\left\{\lambda^{2T_k^*}\right\}$ is

$$\begin{aligned}\mathbb{E}\left\{\lambda^{2T_k^*}\right\} &= (1-p)\lambda^2 + \frac{pq}{(1-q)^2}\frac{(1-q)^2\lambda^4}{1-\lambda^2(1-q)}\\ &= \lambda^2[1+\frac{p(\lambda^2-1)}{1-\lambda^2(1-q)}].\end{aligned}$$

Summarizing the above results, we have

$$\mathbb{E}\left\{\lambda^{2T_k^*}\right\} = \begin{cases}\infty, & \text{if } \lambda^2(1-q)>1\\ \lambda^2\left[1+\frac{p(\lambda^2-1)}{1-\lambda^2(1-q)}\right], & \text{if } \lambda^2(1-q)<1.\end{cases}$$

Then the equivalence of (ii) and (iii) is straightforward from the expression of $\mathbb{E}\left\{\lambda^{2T_k^*}\right\}$.

(i)$\rightarrow$(iii): Let

$$H = Q'D = \begin{bmatrix}1-q & p\delta\\ q & (1-p)\delta\end{bmatrix}.$$

Since H is a nonnegative matrix, in view of Corollary 8.1.20 in [27], $1-q \le \rho(H) < \frac{1}{\lambda^2}$, which is (13). Suppose the two eigenvalues of H are ζ_1 and ζ_2, then $\zeta_1+\zeta_2=\mathrm{tr}(H), \zeta_1\zeta_2=\det(H)$ with $\mathrm{tr}(H)=(1-q)+(1-p)\delta$ and $\det(H)=(1-p-q)\delta$. Since

$$\begin{aligned}\mathrm{tr}(H)^2-4\det(H) &= ((1-q)+(1-p)\delta)^2-4(1-p-q)\delta\\ &= ((1-q)-(1-p)\delta)^2+4pq\delta>0,\end{aligned}$$

we know that the spectral radius of H is

$$\rho(H)=\frac{\mathrm{tr}(H)+\sqrt{\mathrm{tr}(H)^2-4\det(H)}}{2}.$$

Since $\lambda^2\rho(H)<1$, we have that $\lambda^2\sqrt{\mathrm{tr}(H)^2-4\det(H)} < 2-\lambda^2\mathrm{tr}(H)$. Taking square of both sides, we obtain $\lambda^4\det(H)-\lambda^2\mathrm{tr}(H)+1>0$. Substituting the expression of $\mathrm{tr}(H)$ and $\det(H)$ into the above inequality, we have $\lambda^4(1-q-p)\delta-\lambda^2[(1-q)+(1-p)\delta]+1>0$, which implies $\lambda^2\delta[(1-p)-\lambda^2(1-p-q)]<1-\lambda^2(1-q)$. Dividing both sides by $1-\lambda^2(1-q)$, we can obtain (14).

(iii)$\rightarrow$(i): We first note that $\lambda^2\delta<1$ from (14). In view of (13), we further have $2-\lambda^2\mathrm{tr}(H)=1-\lambda^2(1-q)+1-\lambda^2\delta(1-p)>0$. Then (iii)$\rightarrow$(i) can be proved by reversing the proof of (i)$\rightarrow$(iii). The proof is completed. ■

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